PS9-Solution

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Fall 2016

Selected Papers

- Myerson, Roger B.. 1999. "Nash Equilibrium and the History of Economic Theory." *Journal of Economic Literature*, 37(3): 1067-1082.
- Geanakoplos, John. 1992. "Common Knowledge." *Journal of Economic Perspectives*, 6(4): 53-82.
- Brandenburger, Adam. 1992. "Knowledge and Equilibrium in Games." *Journal of Economic Perspectives*, 6(4): 83-101.

Question 1

Problem of the firm:

$$\max_{\{q \in [0,\bar{q}], I \geq 0\}} \; f_0(q, I) = p(q)q - c(I)q - I$$

Question 1

Problem of the planner:

$$\max_{\{q \in [0,\bar{q}], I \geq 0\}} f_1(q, I) = \int_0^q p(y)dy - c(I)q - I$$

Question 1

For fixed q, both objective functions are strictly concave and they differ up to a constant. So the maximizers are singleton and unique: $I_0 = I_1$.

Question 1

Principle of optimality: For each q, the following objective function satisfies SID in q and I, so by Topkis, $I^*(q)$ is strictly increasing in q.

$$I^*(q) = \arg\max \ -c(I)q - I \equiv h(I,q) \Rightarrow \frac{\partial h}{\partial q} = -c(I)$$

which is strictly increasing.

Question 1

$$g(q,t) = \begin{cases} p(q)q - c(I^*(q))q - I^*(q) & \text{t=0} \\ \int_0^q p(y)dy - c(I^*(q))q - I^*(q) & \text{t=1} \end{cases}$$

$$g(q,1) - g(q,0) = \int_0^q p(y)dy - p(q)q$$

Suppose $0 \leq q' \leq q'' \leq \bar{q}$.

$$\int_0^{q''} p(y)dy - p(q'')q'' - \int_0^{q'} p(y)dy - p(q')q'$$

$$> \int_{q'}^{q''} p(y)dy - p(q'')(q'' - q') > p(q'')(q'' - q') - p(q'')(q'' - q') = 0$$

So we have SID in q and t. □

Question 1

The objective function does not satisfy SID in (q, I) and t.
Suppose $q_1 = q_0$ and $I_1 > I_0 \Rightarrow (q_1.I_1) > (q_0, I_0)$.

$$f(q_1, I_1, 1) - f(q_0, I_0, 1) = f(q_1, I_1, 0) - f(q_0, I_0, 0)$$

So it is not strictly increasing in t!

Question 2

Suppose $b_i = x \neq v_i$ and M is the highest bid of other bidders. We need to show $b_i = v_i$ weakly dominates $b_i = x$. Suppose $x > v_i$:

- $M > x$: neither bid wins the auction.
- $M = x$: $b_i = x$ wins the auction and get negative payoff: $v_i - x < 0$.
- $v_i < M < x$: $b_i = x$ wins the auction and get negative payoff: $v_i - x < 0$. $b_i = v_i$ does not win.
- $M = v_i$: $b_i = x$ wins the auction and gets zero payoff, same for $b_i = v_i$.
- $M < v_i$:both bids wins the auction and gets the same payoff: $v_i - M$.

$x < v_i$: Do as exercise!

Question 2

If there are only two bidders:

- $v_1 = v_2 = v$: NE$= \{(b_1, b_2)|b_1 \leq v \leq b_2$ or $b_2 \leq v \leq b_1\}$.
- $v_1 \neq v_2$: WLG $v_1 > v_2$, NE$= \{b_1 \geq v_2,\ b_2 < b_1,\ b_2 \leq v_1\}$.

Question 3

If b_1 strictly dominates b'_1, then the payoff with b_1 is strictly greater no matter what. But if $b_2 > b_1, b'_1$, then the payoff is zero for b_1 and b'_1. So there is not strictly dominant strategy.

Question 3

Any $b_i > v_i$ is weakly dominated by $b_i = v_i$. The latter will end up with zero payoff always, whereas the former will end up with negative payoff if $(b_j) < b_i$ or $(b_j) = b_i$.
$b_i = 0$ is also weakly dominated by $b_i = v_i$.

Question 3

Claim

- If there is a Nash equilibrium, then $b_1 = b_2$.
  proof: If $b_1 \neq b_2$, then WLG, $b_1 > b_2$, then the first player is better off bidding $\frac{b_1+b_2}{2}$.
- There is no Nash equilibrium in this game.
  proof: consider two cases:
  - $b_1 = b_2 > v_2$: player two has 0.5 chance of winning something negative, he has incentive to reduce his bid.
  - $b_1 = b_2 \leq v_2$: player one has incentive to increase her bid to win the auction with probability 1 instead of 0.5 and get the same payoff (marginally).

Question 4

There is no strictly dominated strategies. Let b_i^1 and b_i^2 be two strategies for player i. If player j demand more than 100, then both player i's demands result in zero profit.

Question 4

Any $b_i > 100$ is weakly dominated by $b_i = 100$. The latter will earn payoff of 100 if $b_j = 0$ and zero otherwise, whereas the former earns zero always.

$b_i = 0$ is weakly dominated by $b_i = 100$.

Question 4

We have two sets of Nash equilibrium:

- $\{(b_1, b_2) | b_1 \geq 100,\ b_2 \geq 100\}$: They both earn zero payoff independent of other player's demand and there is no incentive to deviate.
- $\{(b_1, b_2) | b_1, b_2 \geq 0,\ b_1 + b_2 = 100\}$: Given the other player's demand, nobody has incentive to increase her demand since it results in (weakly) less profit and nobody wants to decrease her demand since it will reduce her payoff.