# PS9-Solution

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# Question 1

Let $(\sigma, \mu)$ be a strategy profile where an assessment and $\mu(\cdot|h)$ is a probability distribution on the nodes in the information set $h \in \mathcal{H}$. By consistency, we have completely mixed strategy $\sigma^n$ and assessment $\mu^n$ such that as $n \to \infty$, we have $\mu^n \to \mu$ and $\sigma^n \to \sigma$. Also, $\mu^n$ is derived from $\sigma^n$ using Bayes rule. Any node in the game is reached by a unique path. Let $P(x)$ be the probability of reaching this node when players use $\sigma$ and $P^n(x)$ be the corresponding probability when they play $\sigma^n$. It is required to prove that $\mu(\cdot|h)$ is derived from $\sigma$ using Bayes rule.

# Question 1

I will focus on the nodes that are reached with positive probability when playing $\sigma$. If a node is reached with probability zero, then beliefs can be anything in that information set and Bayes rule is not working. Since $\sigma^n$ is completely mixed, then all nodes are reached with positive probability and beliefs are formed using Bayes rule:

$$\mu^n(x|h) = \frac{P^n(x)}{\sum_{x' \in h} P^n(x')} \quad \forall x \in h$$

$$\therefore n \to \infty, \ \sigma^n \to \sigma \Rightarrow P^n(x) \to P(x) \Rightarrow \mu(x|h) = \frac{P(x)}{\sum_{x' \in h} P(x')}$$

# Question 2

Suppose the assessment $(\sigma, \mu) = ((\sigma_E, \sigma_I), (\mu_E, \mu_I)$ is a sequential equilibrium which mean it should be consistent. This means there exists a sequence $(\sigma^n, \mu^n) = ((\sigma_E^n, \sigma_I^n), \mu^n)$ that converges to the assessment in the limit.
The beliefs $(\mu^n)$ is obtained by Bayes rule:

$$\mu_I^n((i, A)|h_I) = 1 - p_n$$

where $p_n$ is the defined as $\sigma_E^n(h_E) = (p_n, 1 - p_n)$. As $n \to \infty$, we have $\sigma_E(h_E) = (1, 0)$ which means $p_n \to 0$. Therefore $\lim\limits_{n \to \infty} \mu_I^n = 0$ which is not the belief of the original assessment.

# Question 3

Let $(\sigma, \mu)$ be a sequential equilibrium (SE). Since it is a SE, it is sequentially rational on and off equilibrium path. For NE, we only need to be sequentially rational on the equilibrium path, so any SE is a NE. I now have to prove that SE is going to be NE in every Subgame.
Since $(\sigma, \mu)$ is SE, then $(\sigma, \mu)$ is consistent: $\exists(\sigma^n, \mu^n) \to (\sigma, \mu)$ as n goes to infinity and the beliefs are obtained using Bayes rule. So beliefs in the original game is justifiable as we come from a completely mixed strategy close to equilibrium. Now consider an information set $h$ in the original game. If this information set starts a subgame and it is reached in the original game, then it should be part of the $(\sigma, \mu)$, because we get $(\sigma, \mu)$ from a set of completely mixed strategies that put positive probability on every path on and off the equilibrium. So reaching this information set is justifiable and since the original SE is NE, then it will produce a NE in this subgame. So it will be SPNE!