PS8-Solution

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Question 1

Normalize $T(w) = 0$.
Risk neutral: $g(b, 0) = (v - b + w)F(b) + (1 - F(b))w$.
Risk averse: $g(b, 1) = T(v - b + w)F(b) + (1 - F(b))T(w)$.
Need to show that the objective function satisfies SSCP in b.
Let $b'' > b' \geq 0$ and $g(b'', 0) - g(b', 0) \geq 0$.

$$(v - b'')F(b'') - (v - b')F(b') \geq 0 \Rightarrow \frac{F(b')}{F(b'')} \leq \frac{v - b''}{v - b'} \equiv \lambda$$

$$= \frac{\lambda T(v - b' + w) + (1 - \lambda)T(w)}{T(v - b' + w)} < \frac{T(\lambda(v - b') + \lambda w + (1 - \lambda)w)}{T(v - b' + w)}$$

$$= \frac{T(v - b'' + w)}{T(v - b' + w)} \Rightarrow g(b'', 1) - g(b', 1) > 0$$

Question 2

$g(\alpha, t) = \int u(w + \alpha x) f_t(x) dx.$

$$
\begin{aligned}
g_\alpha(\alpha, 1) &= \int u'(w + \alpha x) x \frac{f_1(x)}{f_0(x)} f_0(x) dx \\
&\pm \frac{f_1(0)}{f_0(0)} \int u'(w + \alpha x) x f_0(x) dx \\
&= \int u'(w + \alpha x) [\frac{f_1(x)}{f_0(x)} - \frac{f_1(0)}{f_0(0)}] x f_0(x) dx + \frac{f_1(0)}{f_0(0)} g_\alpha(\alpha, 0) \\
&\Rightarrow g_\alpha(\alpha, 1) > \lambda g_\alpha(\alpha, 0)
\end{aligned}
$$

We have SSCP in α and t. The assertion follows. □

Question 3

$$\pi(p,k) = \max_{p}\ (p-k)d(p)$$

WLG, $p \in [k, \bar{p}) \Rightarrow \dfrac{\partial^2 \pi}{\partial p \partial k} > 0 \Rightarrow$ SID.

So if $k \uparrow$, then $p \uparrow$ and $q \downarrow$.

Question 3

Let $y = p - k$. Then $\pi(y, k) = \max\ yd(y + k)$.

$$\frac{\partial^2 \pi}{\partial y \partial - k} = -d'(y + k) - yd''(y + k)$$

. If $d''(y + k) \leq 0$, then we have SID in y and $-k$.

- $a - bp$: $p^* - k = \frac{a-kb}{p}$, strictly decreasing in k
- e^{a-bp}: $p^* - k = \frac{1}{b}$, independent of k
- ap^{-b}: $p^* - k = \frac{k}{b-1}$, strictly increasing in k

So if demand is concave, we have SID, otherwise we do not know.

Question 4

$$\max_{q\in[0,\bar{q})} Np(q)q - c(Nq) = N(\max_{q\in[0,\bar{q})} p(q)q - \frac{c(Nq)}{N})$$

If $q^*_{N_1} = 0$, then $q^*_{N_2>N_1} \geq 0$. So suppose $q^*_{N_1} > 0$. Take derivative with respect to q from the second component of the objective function:

$$\frac{\partial}{\partial q}(-\frac{c(Nq)}{N}) = -c'(Nq)$$

This derivative of the above expression with respect to N is strictly increasing since $c(\cdot)$ is strictly concave.

Question 5

The average cost is decreasing, $\frac{F}{q} + k$, so we have increasing returns to scale.

Question 5

$$\pi(q) = \begin{cases} 0 & q = 0 \\ (a - bq)q - F - kq & q > 0 \end{cases}$$

If $q^* > 0$, then $q^* = \dfrac{a-k}{2b}$ and $\pi(q^*) = \dfrac{1}{4b}(a-k)^2 - F$. So for $F \geq \dfrac{1}{4b}(a-k)^2$, the profit is zero.

Question 5

$$\pi(q) = \begin{cases} 0 & q = 0 \\ \int_0^q (a - bx)dx - F - kq & q > 0 \end{cases}$$

If $q^* > 0$, then $q^* = \dfrac{a-k}{b}$ and $\pi(q^*) = \dfrac{1}{2b}(a-k)^2 - F$. So for $\dfrac{1}{4b}(a-k)^2 < F < \dfrac{1}{2b}(a-k)^2$, it is socially optimal to produce.

Question 6

Retailer problem:

$$\max_{q}\ (a-bq)q-\bar{p}q \Rightarrow q^{*}=\frac{a-\bar{p}}{2b}$$

Producer problem:

$$\max_{\bar{p}}\ \bar{p}q^{*}-kq^{*} \Rightarrow \bar{p}^{*}=\frac{a+k}{2}$$

Price charged by the producer is $\bar{p}^{*}$, the quantity is q^{*}, the price for consumer is $p^{*}=a-bq^{*}$, the producer's profit is $\frac{(a-k)^2}{8b}$, the retailer's profit is $\frac{(a-k)^2}{16b}$ and the consumer surplus is $\frac{(a-k)^2}{32b}$.

Question 6

If the merger happens:

$$\max\ (a - bq)q - kq \Rightarrow q^* = \frac{a-k}{2b},\ p^* = \frac{a+k}{2}$$

The consumer surplus is $\frac{(a-k)^2}{8b}$ and it is higher since in the previous case, the consumer was affected by two markups on price and after the merger, the producer markup disappears!