PS7-Solution

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Selected Papers

- Machina, Mark J. "Choice Under Uncertainty: Problems Solved and Unsolved." *Journal of Economic Perspectives* 1, no. 1 (1987): 121-54.

Question 1

Define four lotteries as follows:

$l_1 = (0, 1, 0),\ l_1' = (0.1, 0.89, 0.01),\ l_2 = (0, 0.11, 0.89),\ l_2' = (0.1, 0, 0.9)$

Suppose *Independence Axiom* is satisfied and define the following:

$$l_a = (0, 1, 0),\ l_b = (\frac{10}{11}, 0, \frac{1}{11}),\ l_c = (0, 0, 1)$$

$$l_1 = 0.89 l_a + 0.11 l_a,\ l_1' = 0.89 l_a + 0.11 l_b$$

If $l_a \succeq l_b$, then $l_1 \succeq l_1'$.

Question 1

Also

$$l_2 = 0.11 l_a + 0.89 l_c,\ l_2' = 0.11 l_b + 0.89 l_c$$

If $l_a \succeq l_b$, then $l_2 \succeq l_2'$. Thus if $l_1 \succeq l_1'$, then $l_2 \succeq l_2'$.
So choosing $l_2' \succeq l_2$ as many people do would violate the *Independence Axiom*.

Question 2

Let x be the solution at $B(p, w)$ and x' at $B(p, w')$.

$$v(p, w) = u(x) \Rightarrow p \cdot x \leq w$$

$$v(p, w') = u(x') \Rightarrow p \cdot x' \leq w$$

$$\Rightarrow p \cdot (\lambda x + (1 - \lambda)x') \leq \lambda w + (1 - \lambda)w'$$

$$\begin{aligned}\Rightarrow v(p, \lambda w + (1 - \lambda)w') &\geq u(\lambda x + (1 - \lambda)x') \geq \lambda u(x) + (1 - \lambda)u(x') \\ &\geq \lambda v(p, w) + (1 - \lambda)v(p, w')\end{aligned}$$

Question 2

Knowing $v(\cdot)$ is quasiconvex in p:
$v(\lambda p + (1-\lambda)p', w) \leq \max\{v(p,w), v(p',w)\}$.
Take p, p' such that $v(p,w) = v(p',w)$[1].
Thus $v(\lambda p + (1-\lambda)p', w) \leq \lambda v(p,w) + (1-\lambda)v(p',w)$.
So the consumer cannot be strictly risk averse.

[1] We can always find such prices since the dimension of the space is larger than 2 and we cannot have a one-to-one function from $\mathbb{R}^l \to \mathbb{R}$.

Question 2

Risk neutrality:

$$v(\int wdF(w)) = \int v(w)dF(w)$$

This means that $v(\cdot)$ should be an affine function of w, i.e.

$$v(p, w) = a(p) + b(p)w$$

Engel curves are straight lines.

Question 3

$$U(\alpha) = (1-\pi)u(w-\alpha q) + \pi u(w-\alpha q - D + \alpha)$$

If $\alpha = D$, then

$$U'(\alpha) = q(1-\pi)u'(w-\alpha q) + \pi(1-q)u'(w-\alpha q - D + \alpha)$$

$$= u'(w-Dq)(\pi(1-q) - q(1-\pi)) < 0$$

So F.O.C at $\alpha = D$ is negative $\Rightarrow \alpha^* < D$.

Question 4

$$w = m + \alpha \Rightarrow \text{Final wealth} = m + \alpha(1 + x + t) = w + \alpha(x + t).$$

$$g_t(\alpha) = \int u(w + \alpha(x + t))dF(x)$$

$$\frac{dg_t(\alpha)}{d\alpha}|_{\alpha=0,t=0} = 0, \;\; \frac{d^2 g_t(\alpha)}{d\alpha}|_{\alpha=0,t=0} < 0 \Rightarrow \alpha^*(0) = 0$$

$$\frac{dg_t(\alpha)}{d\alpha}|_{\alpha=0,t>0} > 0 \Rightarrow \alpha^*(0) > 0$$

Question 4

F.O.C:$\int (x+t)u'(w+\alpha^*(x+t))dF(x)$.
Using Implicit function theorem as $t \to 0,\ \alpha^* \to 0$:

$$\frac{\partial \alpha^*}{\partial t} = -\frac{\frac{\partial G}{\partial \alpha^*}}{\frac{\partial G}{\partial t}} = -\frac{1}{\frac{u''(w)}{u'(w)}} \cdot \frac{1}{\text{var}(x)} = \frac{1}{r\text{var}(x)}$$

Intuition:

- $r \uparrow$: more risk averse means less inclined to buy risky asset.
- $\text{var}(x) \uparrow$: higher variance of the risk means less inclined to buy risky asset.

Question 5

$$U(x_1, x_2) = \pi_1 u(x_1) + \pi_2 u(x_2)$$

Sum of concave functions is concave, so $U(x_1, x_2)$ is concave, hence it is quasi-concave. Therefore, preferences are convex.

Question 5

Since $U(\cdot)$ is concave, the consumer is risk averse over (x_1, x_2).

Question 5

Since $U(\cdot)$ is additively separable, then both x_1 and x_2 are normal.

Question 6

$$
\begin{aligned}
g(b_1) =& prob(b_1 < b_2)[0 + w] + prob(b_1 > b_2)[v_1 + w - b_1] \\
=& w + prob(b_1 > b_2)[v_1 - b_1] \\
=& w + F(b_1)[v_1 - b_1] \\
\Rightarrow & b_1^* = \arg\max_{b_1}(w + F(b_1)[v_1 - b_1])
\end{aligned}
$$

The bidder never bids more than v_1 since in that case, there is a positive probability of winning something net negative. This means that $b_1 \in [0, v_1)$.

Question 6

From our set up before, we have:

$$\frac{\partial g(b_1)}{\partial v} = F(b_1)$$

Since $F(b_1)$ is strictly increasing[2], $g(b_1)$ is SID in (b_1, v). It is also SPM since it is defined on $\mathbb{R}$. We can apply Topkis and conclude that $v_2 > v_1$, then $b_2 \geq b_1$.

[2] $F'(b_1) > 0$.