PS7-Solution

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Question 1

I assume that the set of mixed strategies for this game is $\{(\mu_i^m, \mu_i^c),\ i = 1, 2\}$.

Player 1 (rows) vs. Player 2 (columns):

	a	b
a	$1, -1$	$-1, 1$
b	$-1, 1$	$1, -1$

Player 1 (rows) vs. Player 2 (columns):

	a	b
a	$1, 1$	$0, 0$
b	$0, 0$	$1, 1$

Figure: From left to right: Both players are type m, both are type c.

Player 1 (rows) vs. Player 2 (columns):

	a	b
a	$1, 1$	$-1, 0$
b	$-1, 0$	$1, 1$

Player 1 (rows) vs. Player 2 (columns):

	a	b
a	$1, -1$	$0, 1$
b	$0, 1$	$1, -1$

Figure: From left to right: Player 1 type m and Player 2 type c, Player 1 type c and Player 2 type m.

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Player 1's expected payoff:

- Playing a when her type is m:
  $u_1(a) = \frac{2}{3}(\mu_2^m - (1 - \mu_2^m)) + \frac{1}{3}(\mu_2^c - (1 - \mu_2^c))$
- Playing b when her type is m:
  $u_1(b) = \frac{2}{3}(-\mu_2^m(1 - \mu_2^m)) + \frac{1}{3}(-\mu_2^c + (1 - \mu_2^c))$

So, if $u_1(a) > u_1(b)$, then Player 1 will choose a with probability 1, if $u_1(a) < u_1(b)$, she chooses b with probability 1 and if $u_1(a) = u_1(b)$, then she is indifferent. So, her best response in case she has a type m is:

$$\mu_1^m = \begin{cases} 1, & 4\mu_2^m + 2\mu_2^c > 3 \\ \in [0, 1], & 4\mu_2^m + 2\mu_2^c = 3 \\ 0, & 4\mu_2^m + 2\mu_2^c < 3 \end{cases}$$

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Similarly, her best response if she has a type c is:

$$\mu_1^c = \begin{cases} 1, & 4\mu_2^m + 2\mu_2^c > 3 \\ \in [0,1], & 4\mu_2^m + 2\mu_2^c = 3 \\ 0, & 4\mu_2^m + 2\mu_2^c < 3 \end{cases}$$

The best response of Player 2 can be calculated by the same logic. Now we have to find probability distributions that are compatible with these best responses.

If $4\mu_2^m + 2\mu_2^c > 3$, then $\mu_1^m = 1,\ \mu_1^c = 1$. Given this, player two would choose $\mu_2^m = 0,\ \mu_2^c = 1$. But this choice violates the condition: $4\mu_2^m + 2\mu_2^c = 2 < 3$. So we can dismiss this case. The case of $4\mu_2^m + 2\mu_2^c < 3$ can be dismissed in a similar way. The only case will be: (μ_i^m, μ_i^c) such that $4\mu_i^m + 2\mu_i^c = 3,\ i = 1, 2$.