PS5-Solution

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Question 1

Let $x_i \in \arg\max f(x, t_i)$, where $i \in \{1, 2, \lambda \in [0, 1]\}$.

$$f(x_1, t_1) \geq f(x_\lambda, t_1), \qquad f(x_2, t_2) \geq f(x_\lambda, t_2) \Rightarrow$$

$$\begin{aligned}
\lambda v(t_1) + (1-\lambda) v(t_2) &= \lambda f(x_1, t_1) + (1-\lambda) f(x_2, t_2) \\
&\geq \lambda f(x_\lambda, t_1) + (1-\lambda) f(x_\lambda, t_2) \\
&\geq f(x_\lambda, \lambda t_1 + (1-\lambda) t_2) \\
&= v(\lambda t_1 + (1-\lambda) t_2)
\end{aligned}$$

Question 2

Suppose $\mathbb{Y}$ is a convex set. Let $q^i = f(z^i)$ for $i \in \{1, 2\}$. This means that $(-z^i, q^i) \in \mathbb{Y}$. Since $\mathbb{Y}$ is convex, we have $(-z_\lambda, q_\lambda) \in \mathbb{Y}$ where $z_\lambda = \lambda z^1 + (1-\lambda)z^2$ and analogously for q_λ. By definition of the production function:

$$f(z_\lambda) \geq q^\lambda = \lambda f(z^1) + (1-\lambda) f(z^2)$$

Thus f is concave.

Question 3

By definition: $y^i \in \arg\max \; p^i \cdot y$. It follows that

$$\begin{aligned} p^1 \cdot y^1 \geq p^1 \cdot y^0, \quad p^0 \cdot y^0 \geq p^0 \cdot y^1 \Rightarrow \\ p^1 \cdot y^1 + p^0 \cdot y^0 \geq p^1 \cdot y^0 + p^0 \cdot y^1 \\ (p^1 - p^0) \cdot (y^1 - y^0) \geq 0 \end{aligned}$$

When just one price changes, if the good is output, we get the supply law. If it is input ($-z \uparrow \Rightarrow z \downarrow$), we get the demand law for inputs.

Question 4

$$\max_{\{x,n,h\}} \; pq(x,n,h) - nhw - nb - r \cdot x$$

Revealed preferences:

$$pq(x^1,n^1,h^1) - n^1h^1w - n^1b^1 - r \cdot x^1 \geq$$
$$pq(x^0,n^0,h^0) - n^0h^0w - n^0b^1 - r \cdot x^0$$
$$pq(x^0,n^0,h^0) - n^0h^0w - n^0b^0 - r \cdot x^0 \geq$$
$$pq(x^1,n^1,h^1) - n^1h^1w - n^1b^0 - r \cdot x^1$$

Summing up both sides and canceling the repeated terms , we get:

$$(n^1 - n^0)(b^1 - b^0) \leq 0$$
$$(n^1h^1 - n^0h^0)(w^1 - w^2) \leq 0$$

Question 5

The objective function does not have SID or SSCP in $-w_1$ and $(z_1, ..., z_n)$.

Question 5

From revealed preferences argument in Question 3, we know that $z_1^1 \geq z_1^0$. So we have shown that $\underline{z}_1^1 \geq \underline{z}_1^0$. Now we use the *Principle of Optimality.* Consider the following problem:

$$\max_{\{z_2,...,z_n\}} \; pf(\underline{z}_1, z_2, ..., z_n) - w_1\underline{z}_1 - w_2 z_2 - ... - w_n z_n$$

The objective function is SID in $\underline{z}_1$ and $(z_2, ..., z_n)$ and SPM in $(z_2, ..., z_n)$. If $\underline{z}_1^1 > \underline{z}_1^0$, then by Topkis we can say that $(z_2^1, ..., z_n^1) \geq (z_2^0, ..., z_n^0)$. Consider two cases:

1. $\underline{z}_1^1 = \underline{z}_1^0 \Rightarrow (\underline{z}_2^1, ..., \underline{z}_n^1) = (\underline{z}_2^0, ..., \underline{z}_n^0)$
2. $\underline{z}_1^1 > \underline{z}_1^0 \Rightarrow (\underline{z}_2^1, ..., \underline{z}_n^1) \geq (\underline{z}_2^0, ..., \underline{z}_n^0)$