PS4-Solution

Mehrdad Esfahani

Arizona State University

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Question 1

Choose some $1 \leq k \leq l$ and fix the level of consumption of the goods index by $i \geq k$ is fixed and choose two bundles with different levels of all other goods: x, y

$$x \succeq y \Leftrightarrow u(x) \geq u(y) \Leftrightarrow$$

$$\sum_{i=1}^{k} u_i(x_i) + \sum_{i=k+1}^{l} u_i(\bar{x}_i) \geq \sum_{i=1}^{k} u_i(y_i) + \sum_{i=k+1}^{l} u_i(\bar{x}_i) \Leftrightarrow$$

$$\sum_{i=1}^{k} u_i(x_i) \geq \sum_{i=1}^{k} u_i(y_i)$$

Question 1

Let $w_1 > w_0$ and for $t = 0, 1$ let x^t solves the utility maximization problem. Rtp: $x^1 \geq x^0$. If $x_i^0 = 0$, then obviously $x_i^1 \geq x_i^0$. So let's assume $x_i^0 > 0$. Since $w_1 > w_0$, $x_l^1 \geq x_l^0$ for some good l. Let λ_i be the marginal utility of wealth level i. we have:

$$\frac{u_i'(x_i^1)}{p_i} \leq \lambda_1 = \frac{u_l'(x_l^1)}{p_l} < \frac{u_l'(x_l^0)}{p_l} \leq \lambda_0 = \frac{u_i'(x_i^0)}{p_i}$$

Since $u_i'(\cdot)$ is strictly decreasing, $x_i^1 > x_i^0$.

Question1

For compensated demand: $h(p, v(p, w)) = d(p, w)$. If w increases, $v(p, w)$ increases because of LNS and hence compensated demand increases.

Question 1

If good l is not the one with non-concave function, then the analysis is exactly the same as the previous part. If one the other hand, good l is the one that increases, then depending on the sign of the derivative of the marginal utility, other goods are either all inferior locally (assuming consumption of each is positive of course). So we must have some inferior good in this case.
An example of such function is: $u(x_1, x_2) = -e^{-x_1} + e^{x_2}$

Question 1

Example

Complementary goods: computer, mouse, keyboard
Habit formation: the utility of dinner depends on what was eaten for lunch and breakfast.

Question 2

By Walras' law, $\alpha p \cdot x(\alpha p, w) \leq \alpha w$, so that $p \cdot x(\alpha p, w) \leq w$. By the weak axiom, if $x(p, w) \neq x(\alpha p, \alpha w)$, then $\alpha p \cdot x(p, w) > \alpha w$, so that $p \cdot x(p, w) > w$, which is a violation of Walras' law. Therefore, $x(p, w) = x(\alpha p, \alpha w)$.

Question 2

For this part, we need two expressions that I leave their proof as an exercise:

$$S(p,w) \cdot p = 0 \qquad S^T(p,w) \cdot p = 0$$

For the two good case, we have

$$\begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \cdot \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} s_{11} & s_{21} \\ s_{12} & s_{22} \end{bmatrix} \cdot \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Therefore, $p_1 s_{11} + p_2 s_{12} = p_1 s_{11} + p_2 s_{21} = 0 \Rightarrow s_{12} = s_{21}$.

Question 3

Using Roy's identity:

$$d_l^i(p, w^i) = -\frac{\dfrac{\partial a^i(p)}{\partial p_l} + \dfrac{\partial b(p)}{\partial p_l} \cdot w^i}{b(p)}$$

For this being homothetic, we need the demand to be HD1 in w. Therefore $a^i(p)$ is a constant. For it to be quasilinear in good l, $b(p)$ should be a function of p_l.[1]

[1] Any change in prices rather than p_l should not change the demand and the wealth effect disappears.

Question 3

Using the fact that $v(p, e(p, u)) = u$, we can find the expenditure function:

$$e(p, u) = \frac{u - a(p)}{b(p)}$$

We do not know anything about the curvature of consumer's problem. So no inference about the expenditure function can be done.

Question 3

I propose the following redistribution:

$$\tilde{w}^i = \frac{a^i(p) - a^i(q)}{b(q)} + \frac{b(p)}{b(q)} w^i$$

Summing up over all consumer to check that if this redistribution is feasible:

$$\sum_{i=1}^{I} \tilde{w}^i - w^i = \frac{1}{b(q)} (V(p, w) - V(q, w)) \leq 0$$

From the redistribution equation, we have:

$$a^i(q) + b(q)\tilde{w}^i \geq a^i(p) + b(p) w^i$$

This means everyone is weakly prefer this redistribution.

Question 4

The indifference between the prices means $p_1 p_2 = q_1 q_2$. Suppose we have a wealth redistribution to make consumer 1 indifferent, then $\tilde{w}_1 = \frac{q_1}{2p_1}$. Doing the same for consumer 2, $\tilde{w}_2 = \frac{q_2}{2p_2}$. For feasibility of this redistribution, we need
$\tilde{w}_1 + \tilde{w}_2 \leq 1 \Rightarrow (q_1 - p_1)^2 \leq 0.$[2]
So, there is no redistribution.

[2]Keeping in mind that $p_1 p_2 = q_1 q_2$

Question 5

Since $0 \in \mathcal{Y}$, then $\pi(p) \geq p \cdot 0 = 0$. Consider two possibilities:

1. $\forall y \in \mathcal{Y}: \; p \cdot y \leq 0 \Rightarrow \pi(p) = 0$.
2. $\exists y_0 \in \mathcal{Y}$ such that $p \cdot y > 0 \Rightarrow \beta y \in \mathcal{Y}, \; \forall \beta \geq 1$
   $\Rightarrow \beta y_0 \in \mathcal{Y} \Rightarrow \pi(p) \to +\infty$