PS4-Solution

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Question 1

Assuming interior solutions, we have:

$$x_1 = p_1 f'(k_1),\ x_1' = p_1' f'(k_1')$$

Suppose $p_1' > p_1$ and good 1 uses input 1 more intensely. From Stopler-Samulson Theorem, we have:

$$x_1' > x_1,\ x_2' < x_2 \Rightarrow \frac{x_2'}{x_1'} < \frac{x_2}{x_1}$$

From the conclusion in the lectures, the more intense factor is increasing in the factor prices $\frac{x_2}{x_1}$. This means that $k_1' < k_1$. Using the fact that $f(\cdot)$ is increasing and concave, we have $f'(k_1) < f'(k_1')$:

$$\frac{x_1'}{x_1} = \frac{p_1'}{p_1} \cdot \frac{f'(k_1')}{f'(k_1')} \Rightarrow \frac{x_1'}{x_1} > \frac{p_1'}{p_1}$$

Question 2-Part a

The equilibrium is a set consisting of vectors $(p^*, w^*, x_1^*, x_2^*, z_1^*, z_2^*)$ such that:

- Given (p^*, w^*), x_i^* solves the consumer's problem for each $i = 1, 2$:

$$\max_{x_i \geq 0} \; u_i(x_i) \tag{1}$$

$$s.t. \; p^* \cdot x_i \leq w_i^*$$

- Given (p^*, w^*), z_j^* solves the firm's problem for each $j = 1, 2$

$$\max_{z_i \geq 0} \; p_j^* f_j(z_j) - w^* \cdot z_j \tag{2}$$

- Markets Clear:

$$\sum_i^2 x_i^* = (f_1(z_1^*), f_2(z_2^*)), \; \sum_j^2 z_j^* = (1, 1) \tag{3}$$

Question 2-Part b

Suppose we have two equilibria. Suppose further that $p_2 = p'_2 = 1$ and $p'_1 > p_1$. From question 1, we have:

$$\frac{w'_1}{w_1} > \frac{p'_1}{p_1}$$

Optimality conditions:

$$f_2(z_2) = w_1,\ p_1 f_1(z_1) = w_2,\ f_2(z'_2) = w'_1,\ p'_1 f_1(z'_1) = w'_2$$

$$\Rightarrow \frac{f_2(z'_2)}{f_2(z_2)} > \frac{p'_1}{p_1} > 1 \Rightarrow f_2(z'_2) > f_2(z_2) \Rightarrow z'_2 > z_2$$

This is counterintuitive since the price of capital has gone up (due to SS theorem), but the less intensive firm while facing the same product price is using capital more.

Question 2-Part c

We can follow the same logic and construct two different equilibia with no contradiction. The optimality conditions are:

$$f_2(z_2) = w_2,\ p_1 f_1(z_1) = w_1,\ f_2(z_2') = w_2',\ p_1' f_1(z_1') = w_1'$$

Using SS theorem, we have:

$$\frac{f_2(z_2')}{f_2(z_2)} > \frac{p_1'}{p_1} \Rightarrow \frac{p_1' f_1(z_1')}{p_1 f_1(z_1)} > \frac{p_1'}{p_1} \Rightarrow f_1(z_1') > f_1(z_1)$$

$$w_2' < w_2 \Rightarrow f_2(z_2') < f_2(z_2)$$

The equilibrium prices and allocations do not contradict the two equilibria along with theorems.

Question 3

I choose the functions u_i to be constant functions since all they should be is nondecreasing. Set $u_i = 0 \; \forall \; i$.
Also, $w(\cdot)$ can be arbitrary, so set that one also to constant function with zero value. We get $v_i = 0 \; \forall \; i$.
If we choose $s, s' \in [0, w]^n$ with $p(s' - s) \neq \phi$, then the following maximizations would not hold the property of dominant effect:

$$\max_{i \in p(s'-s)} v_i(s') - v_i(s) = 0$$

$$\max_{j \notin p(s'-s)} v_j(s') - v_j(s) = 0$$

The first one can never be strictly larger than the second one.
Same result can be obtained by setting u_is to any constant and $w(\cdot)$ to be any constant function.