PS3-Solution

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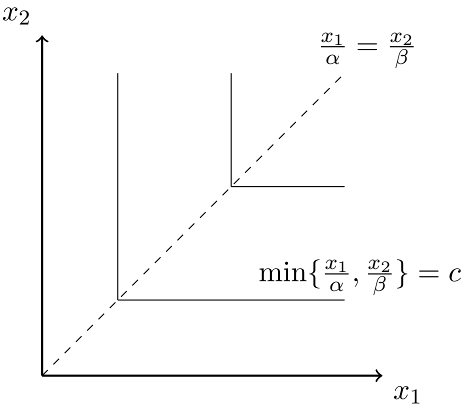
Question 1


x_2
$\frac{x_1}{\alpha} = \frac{x_2}{\beta}$
$\min\{\frac{x_1}{\alpha}, \frac{x_2}{\beta}\} = c$
x_1

Question 1

$$u(x_1, x_2) = min\{\frac{x_1}{\alpha}, \frac{x_2}{\beta}\}$$

$$d_1(p_1, p_2, w) = \frac{\alpha w}{\alpha p_1 + \beta p_2}$$

$$d_2(p_1, p_2, w) = \frac{\beta w}{\alpha p_1 + \beta p_2}$$

$$v(p_1, p_2, w) = \frac{w}{\alpha p_1 + \beta p_2}$$

$$h_1(p_1, p_2, w) = \alpha u, \quad h_1(p_1, p_2, w) = \beta u$$

$$e(p_1, p_2, w) = (\alpha p_1 + \beta p_2) u$$

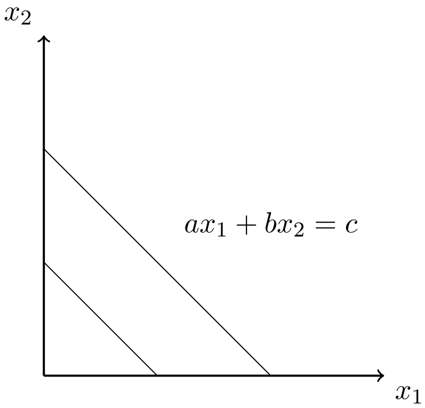
Question 1

For determining the quasiconvexity of the indirect utility function, we can take logs and look at the Hessian matrix:

$$H = \frac{1}{(\alpha p_1 + \beta p_2)^2} \begin{bmatrix} \alpha^2 & \alpha\beta \\ \alpha\beta & \beta^2 \end{bmatrix}$$

The matrix in positive semi-definite, so $\log v(\cdot)$ is convex and $v(\cdot)$ is quasiconvex in (p_1, p_2). The expenditure function is linear and hence concave.

Question 1



Question 1

$$u(x_1, x_2) = ax_1 + bx_2$$

$$d(p_1, p_2, w) = \begin{cases} (\frac{w}{p_1}, 0) & \frac{a}{b} > \frac{p_1}{p_2}; \\ \\ \{(\alpha, \frac{w - p_1\alpha}{p_2}) | \alpha \in [0, \frac{w}{p_1}]\} & \frac{a}{b} = \frac{p_1}{p_2}; \\ \\ (0, \frac{w}{p_2}) & \frac{a}{b} < \frac{p_1}{p_2}. \end{cases}$$

$$v(p_1, p_2, w) = \max\{\frac{aw}{p_1}, \frac{bw}{p_2}\}, \quad e(p_1, p_2, w) = \min\{p_1\frac{u}{a}, p_2\frac{u}{b}\}$$

$v(\cdot)$ is max over convex functions, so it is convex and quasiconvex. $e(\cdot)$ is min over concave functions, so it is concave.

Question 1

$$h(p_1, p_2, u) = \begin{cases} (\frac{u}{a}, 0) & \frac{a}{b} > \frac{p_1}{p_2}; \\ \{(\alpha, \frac{u - \alpha a}{b}) | \alpha \in [0, \frac{u}{a}]\} & \frac{a}{b} = \frac{p_1}{p_2}; \\ (0, \frac{u}{b}) & \frac{a}{b} < \frac{p_1}{p_2}. \end{cases}$$

Question 2

The following utility function which is not continuous, monotone, LNS and convex generates the mentioned demand functions.

$$u(x_1, x_2) = 1, \quad \text{if } x_1 = \frac{x_2}{2} \text{ and zero elsewhere.}$$

Question 3

Since preferences are monotone, the budget constraint binds and we can calculate the demand for good 3.

$p_1x_1 + p_2x_2 + p_3x_3 = w$

The demands are homogeneous of degree 0.

Question 3

Utility maximization implies that the Slutsky Matrix is symmetric: $s_{12} = s_{21}$

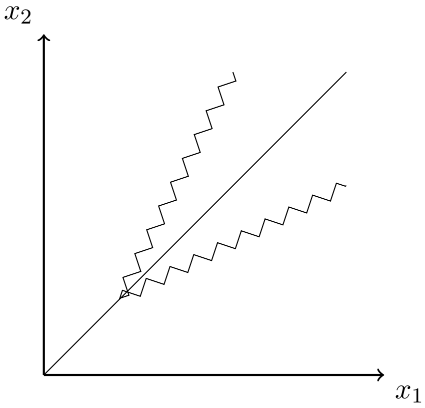
$$s_{ij} = \frac{\partial x_i}{\partial p_j} + \frac{\partial x_i}{\partial w} \cdot x_i$$

This gives us the proper restrictions:

$\alpha = 100,\ \beta = -5,\ \gamma = -5,\ \delta > 0.$

Question 3

By monotonicity and convexity, we rule out weird cases such as below. We also need continuity.



Question 3

There is no wealth effects on the demand for good 1 and 2. So we have a quasilinear utility function:

$$u(x_1, x_2, x_3) = f(\min\{x_1, x_2\}) + x_3$$

or a monotone transformation of this function. To pin down the functional form, note that we know $x_1 = x_2$. Call this common value y and set up the maximization problem:

$$\max \; f(y) + x_3 \quad s.t \; (p_1 + p_2)y + p_3 x_3 = w$$

The F.O.C gives us: $f'(y) = \frac{p_1+p_2}{p_3}$. Using the demand functions and substituting for y and integrating back, we can find the functional form: $f(y) = -\dfrac{y^2}{10} + 20y + c$.

Question 4

$$v(p, \bar{w}) = \max_{x_1, x_2} \; u(x_1, x_2)$$

$$s.t. \quad p_1 x_1 + p_2 x_2 \leq p_1 w_1 + p_2 w_2 = \bar{w}$$

It is easy to check that the indirect utility function is HD0 in prices.

For the Roy's Identity, we have:

$$\frac{\partial v(p, p \cdot w)}{\partial p_i} = \frac{\partial v(p, \bar{w})}{\partial p_i} + \frac{\partial v(p, \bar{w})}{\partial \bar{w}} \cdot w_i$$

$$\Rightarrow x_i(p, \bar{w}) = \frac{\dfrac{\partial v(p, \bar{w})}{\partial p_i}}{\dfrac{\partial v(p, \bar{w})}{\partial \bar{w}}} + w_i$$

Question 2

$$\frac{\partial x_i(p, p \cdot w)}{\partial p_i} = \frac{\partial x_i(p, \bar{w})}{\partial p_i} + \frac{\partial x_i(p, \bar{w})}{\partial \bar{w}} \cdot w_i$$

Slutsky Equation:

$$\frac{\partial x_i(p, \bar{w})}{\partial p_i} = \frac{\partial h_i(p, \bar{u})}{\partial p_i} - \frac{\partial x_i(p, \bar{w})}{\partial p_i} \cdot x_i(p, \bar{w})$$

$$\frac{\partial x_i(p, p \cdot w)}{\partial p_i} = \frac{\partial h_i(p, \bar{u})}{\partial p_i} + \frac{\partial x_i(p, \bar{w})}{\partial p_i}(w_i - x_i(p, \bar{w}))$$

Question 4

$\frac{\partial x_i(p, p \cdot w)}{\partial p_i}$ can be positive or negative. So both statements are wrong.

Question 4

$p \cdot x \leq p_1(w_1 - 3) + p_2(w_2 + 4)$. For this to be profitable, we need $-3p_1 + 4p_2 \geq 0$.

$$W(p) = \max_{w \in C} \; p \cdot w$$

We do not need to know about the preferences in order to find the optimal endowment choice.