PS2-Solution

Mehrdad Esfahani

Arizona State University

Spring 2017

Question 1

Proof by contradiction:
Suppose $(\hat{x}_a, \hat{x}_b)$ is a feasible PO allocation which is not a solution to the problem. Set $\bar{U}_b = U_b(\hat{x}_b)$, which can be done given the continuity and strong monotonicity of the utility functions. Suppose (x_a, x_b) is a solution to the maximization problem. Then we have: $U_a(x_a) > U_a(\hat{x}_a)$ and $U_b(x_b) \geq \bar{U}_b = U_b(\hat{x}_b)$. This means that $(\hat{x}_a, \hat{x}_b)$ is not PO, which is a contradiction!

Question 2-Part a

$$Y = \{(x, y)|x \leq 0,\ y \leq (-x + a)^{0.5}\}$$

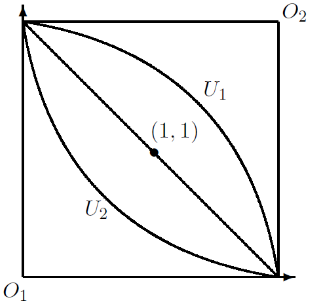
Question 2-Part b

We can use the second welfare theorem (why?) and solve the planner's problem:

$$\max_{\{x_1,x_2,y\}} \log x_1 + \log y,\ y \leq (x_2+a)^{0.5}+1,\ x_1+x_2=1$$

if $x_1 < 1$, the FOC binds and we have: $x_1 = \frac{3a+2+(3a+2)^{0.5}}{0.5a}$, $y = (1-x_1+a)^{0.5}, p = \frac{y}{x}$. if $x_1 = 1$, then $y = a^{0.5}+1$ and $\frac{1}{2a^{0.5}} \leq p \leq 1+a^{0.5}$.

Question 3-Part a



WE: $x_1^1 = 2, x_2^1 = 0, x_1^2 = 0, x_2^2 = 2, p_1 = p_2 = 1$
$x_1^1 = 0, x_2^1 = 2, x_1^2 = 2, x_2^2 = 0, p_1 = p_2 = 1$

Question 3-Part b

WE: $p_1 = p_2 = 1$. If I is even, half the individuals consume (2,0) and the other half consume (0,2). If I is odd, there is no WE.

Question 3-Part c

The corner solutions with $p_1 = p_2 = 1$ are (1,0) and (0,1) for consumer one and (3,0) and (0,3) for consumer two which are not feasible given that the total endowment of each good is 2!

Question 4-Part a

Let p, x be the WE price and allocation. Strict quasiconcavity means single-valued demand. HD1 means demand is HD1 in w. This means the demand for consumer i and good l is:

$$x_l^i(p, p \cdot w^i) = (p \cdot w^i) x_l^i(p, 1) = (p \cdot w^i) x_l^i(p)$$

Since consumers are all the same we have:
$x_l^i(p) = x_l^j(p) = x_l(p) \; \forall \; i, j$. Market clearing condition for good l:

$$\sum_{i=1}^{I} x_l^i(p, p \cdot w^i) = \sum_{i=1}^{I} (p \cdot w^i) x_l(p) = x_l(p) \sum_{i=1}^{I} p \cdot w^i =$$

$$\sum_{i=1}^{I} w_l^i \Rightarrow x_l(p) = \frac{\sum_{i=1}^{I} w_l^i}{\sum_{i=1}^{I} p \cdot w^i}, \; \frac{p_k}{p_l} = MRS_{k,l}^i$$

Question 4-Part b

Both welfare theorems hold (why?), so the WE is PO.
From the previous part, we have:

$$x_l^i(p, p \cdot w^i) = \frac{(p \cdot w^i) \sum_{i=1}^{I} w_l^i}{\sum_{i=1}^{I} p \cdot w^i} = \alpha_i \sum_{i=1}^{I} w_l^i.$$

For $I = 2$, we have: $\alpha_1 + \alpha_2 = 1$. So by moving α_1 from 0 to 1, we get all the PO allocations.

Question 4-Part c

Apply the following strictly increasing transformation: $f(x) = x^{\frac{1}{k}}$. The resulting function is HD1 and all the results follow.

Question 5

Apply log transformation. Let $p_x = 1$. Consumer A's problem is:

$$\max_{\{x_a, y_a, z_a\}} \log x_a + \log y_a + 2\log z_a \quad s.t.\ x_a + p_y y_a + p_z z_a = 2 + p_y + 3p_z$$

Find the allocations in terms of prices:

$$x_a = \frac{2 + p_y + 3p_z}{4},\ y_a = \frac{2 + p_y + 3p_z}{4p_y},\ z_a = \frac{2 + p_y + 3p_z}{2p_z}$$

Do the same for consumer B. Then use the feasibility constraint to find prices.

$$x_a = x_b = 2,\ y_a = 4/3, y_b = 8/3,\ z_a = 8/3, z_b = 4/3$$

Question 6-Part a

PO allocations:

$$\max\ \alpha_1(\log x_1 + \log y_1) + (1-\alpha_1)(0.5\log x_2 + \log y_2)$$

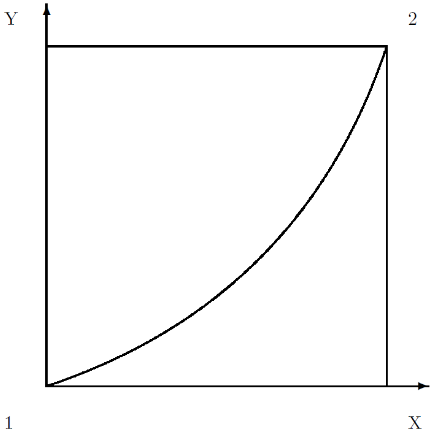
$$s.t.\ x_1 + x_2 = 1,\ y_1 + y_2 = 1$$

The solution will be:

$$x_1 = \frac{2\alpha_1}{2\alpha_1 + \alpha_2},\ x_2 = 1 - x_1$$
$$y_1 = \frac{\alpha_1}{\alpha_1 + \alpha_2},\ y_2 = 1 - y_1$$

Can there be corner solutions?

Question 6-Part a



Question 6-Part a

WE: Follow the usual steps:

$$x_1 = \frac{3}{5},\ x_2 = \frac{2}{5},\ y_1 = \frac{3}{7},\ y_2 = \frac{4}{7},\ \frac{p_y}{p_x} = \frac{7}{5}$$

Question 6-Part b

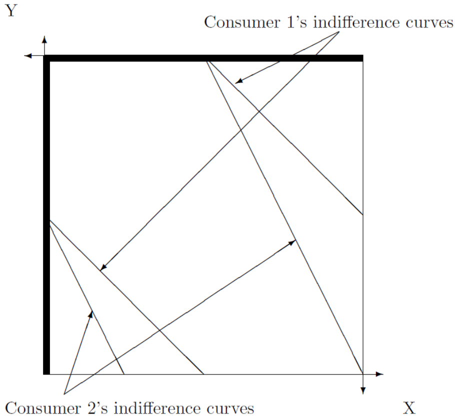
PO allocations:

$$\max \ \alpha_1(x_1 + y_1) + (1 - \alpha_1)(2x_2 + y_2)$$

$$s.t. \ x_1 + x_2 = 1, \ y_1 + y_2 = 1$$

We only get corner solutions!

Question 6-Part b



Question 6-Part b

WE: Follow the usual steps and note that we only have corner solutions:

$$x_1 = 0,\ x_2 = 1,\ y_1 = 1,\ y_2 = 0,\ \frac{p_y}{p_x} = 1$$

Question 6-Part c

PO allocations:

$$\max\ \alpha_1(\log x_1 + y_1) + (1-\alpha_1)(\log x_2 + 2y_2)$$

$$s.t.\ x_1 + x_2 = 1,\ y_1 + y_2 = 1$$

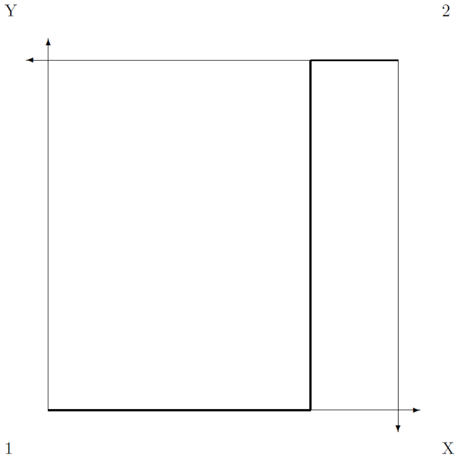
We have both corner and interior solutions!

$$x_1 = \frac{2}{3},\ x_2 = \frac{1}{3},\ y_1 + y_2 = 1$$

$$0 < x_1 < \frac{2}{3},\ \frac{1}{3} < x_2 \leq 1,\ y_1 = 0,\ y_2 = 1$$

$$\frac{2}{3} < x_1 \leq 1,\ 0 < x_2 < \frac{1}{3},\ y_1 = 1,\ y_2 = 0$$

Question 6-Part c



Question 6-Part c

WE: Follow the usual steps:

$$x_1 = \frac{2}{3},\ x_2 = \frac{1}{3},\ y_1 = 0.5,\ y_2 = 0.5,\ \frac{p_y}{p_x} = \frac{2}{3}$$