PS11-Solution

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Fall 2016

Question 1

Any production below $\underline{Q}$ or $\bar{Q}$ is not efficient since price is less than the average cost in those regions. In the interval $[\underline{Q}, \bar{Q}]$, we have a Bertrand game and the production will be $\bar{Q}$ and NE is (p^*, p^*). As a result, firm 1 serves the whole market.
Both firms get zero profit. Each firm's profit (payoff) is not continuous in its own price (strategy), but quasiconcave in other firm's price.

Question 1

Planner can pick only one firm to operate, since any firm can provide for the whole market and there is a fixed cost of operation. planner solves the following maximization:

$$\max_{q \in [\underline{Q}, \bar{Q}]} \int_0^q p(x)dx - F - kq$$

First order condition gives: $p(q) - k$. It monotonically decreases over the $[\underline{Q}, \bar{Q}]$. So the solution will be $p(\bar{Q})$. There is no difference in welfare compared to previous part other than the fact that planner can pick firm 2 to operate.

Question 2

Producer strategy set: $s_M = \{p|p \geq 0\}$.
Retailer strategy set: $s_R = \{q_R|q_R : \mathbb{R}_+ \rightarrow \mathbb{R}_+\}$. The one we solved before was SPNE, since we used backward induction to solve it. $SPNE = \{(\frac{a+k}{2}, q_R(p) = \frac{a-p}{2b})\}$.
Another NE=$(\frac{a}{2b}, q_R(p) = \frac{a}{2})$.
Producer charges monopoly price and gets the maximum profit. Retailer gets zero profit, which is optimal given producer strategy. Neither has incentive to deviate.

Question 3

In the second stage with one firm, it charges $\frac{a}{2}$, if there are two firms, the price they both charge is 0.
$s_i = \{(a, x, y) | a \in \{\text{In,Out}\}, x, y \in \mathbb{R}_+\}$, where x is for when firm i is alone in the market and y when not alone.
SPNE=$\{[(In, \frac{a}{2}, 0), (Out, \frac{a}{2}, 0)], [(Out, \frac{a}{2}, 0), (In, \frac{a}{2}, 0)]\}$. The second stage subgame strategies are clearly NE. If the firm that enters the market does not enter, it loses a positive profit. If the firm that did not enter decides to enter, it gets -k. So there is no incentive to deviate.

Question 3

SPNE=$\{(In, \frac{a}{2}, \frac{a}{3}), (In, \frac{a}{2}, \frac{a}{3})\}$.
The second stage strategies are NE of Cournot game. In the first stage, no firm would want to quit and lose the positive profit.

Question 3

NE=$\{(In, \frac{a}{2}, a), (Out, \frac{a}{2}, 0)\}$.
First firm gets the most profit. Firm 2 will never enter since it gets negative profit if produces any amount. So none has incentive to deviate.
This is not SPNE since it does not induce a NE in the second stage where both firms are in the market.

Question 4

player i has k_i strategies.

Question 4

Player 1 has k_1 strategies. Player 2 has $k_2^{k_1}$ strategies.

Question 4

If for different strategies, payoffs are always different, then players can rank them (since the strategy set is finite), so backward induction will result in a unique SPNE.

Question 4

Player 2 has a unique best response at each of her decision nodes. Let the action profile that gives player 1 the payoff π_1 be (m_1^*, m_2^*). Player 1 can always get at least π_1 by choosing m_1^*, since player 2 will always respond with m_2^*. So player 1 will never choose something that gives her less that π_1.
This is not true for every NE.

Question 5

$S_1 = \{iabc | i \in \{0, 1, 2\}, a, b, c \in \{U, D\}\}$. Example: $S_1 = 0UUU$.
$S_2 = \{abc | abc \in L, M, R\}$. Example: $S_2 = LLL$.

Question 5

$$NE = \{(0DDD, MRR)\}$$

Question 5

For unique SPNE, we must have unique NE in the second stage. We can ignore the payoff of the first stage (why?), not that (D, M) is a NE for the simultaneous game independent of x. So (U, L) must not be a NE $\Rightarrow x < 0$.

Question 5

If $x + 0 \geq 1 + 2$, the $(0UDD, LMM)$ is a SPNE.

Question 5

Independent of x, player 1 never gets negative payoff in the second stage in any NE. So in any strategy profile in which player 1 gets less than 2 and plays a NE in the second stage, she can always deviate and plays $z = 2$ and gets 2 for sure instead of less than 2. So in any SPNE, player 1 gets at least 2.