PS10-Solution

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Question 1

Depending on the values of a and b, we have different equilibria. Suppose player 2 mixes with $(q, 1-q)$. Then player 1 responds

$$r_1(q, 1-q) = \arg\max_{p \in [0,1]} p(bq) + (1-p)((1-q)a) = \begin{cases} 1 & q > \frac{a}{a+b} \\ 0 & q < \frac{a}{a+b} \\ [0,1] & q = \frac{a}{a+b} \end{cases}$$

Same for player 2.

$$NE(\Delta G) = \{((1,0),(1,0)), ((\frac{b}{a+b}, \frac{a}{a+b}), (\frac{a}{a+b}, \frac{b}{a+b})), ((0,1),(0,1))\}$$

Question 2

Choosing M is best since it has minimum risk.

Question 2

Pure strategy Nash equilibrium: $\{(u, LL), (D, R)\}$.
If $\pi_U > 0 \Rightarrow \pi_{LL} > 0$.
If $\pi_D > 0 \Rightarrow \pi_L + \pi_M + \pi_R > 0$.
$\pi_{LL} > 0 \Rightarrow 2\pi_U - 100\pi_D \geq 0 \Rightarrow \pi_R = 0$.
But then, $\pi_U - 49\pi_D > 0 \Rightarrow \pi_M = 0$.
$2\pi_U - 100\pi_D = \pi_U - 49\pi_D$, which gives us the result.
Mixed strategy Nash equilibrium:
$\{((1,0),(1,0,0,0)),((0,1),(0,0,0,1)),((\frac{51}{52},\frac{1}{52}),(\frac{1}{2},\frac{1}{2},0,0))\}$.

Question 3

In Nash equilibrium, M is played with probability zero. But it is rationalizable since it is a best response to player 1 playing (0.5,0.5).
Playing D is a best response to player 2 playing R.

Question 2

If they communicate, they agree on one of the pure strategies.

Question 3

$$\pi_1(s_1, s_2) = \begin{cases} -s_1 & s_1 < s_2 \\ \frac{1}{2} - s_1 & s_1 = s_2 \\ 1 - s_1 & s_1 > s_2 \end{cases}$$

Question 3

There is no pure strategy Nash equilibrium.

- $s_1 < s_2$: player 2 will be better off by choosing $\frac{s_1+s_2}{2}$.
- $s_2 < s_1$: player 1 will be better off by choosing $\frac{s_1+s_2}{2}$.
- $s_1 = s_2 < 1$: player 1 will be better off by choosing $s_1 + \epsilon$.
- $s_1 = s_2 = 1$: player 1 will be better off by choosing 0.

It is easy to see that π_1 is not continuous. Let $s_2 \in (0, 0.5)$. If $s_1 = 0$, then $\pi_1(s_1, s_2) = 0$. If $s_1' = s_2$, then $\pi_1(s_1', s_2) = 0.5 - s_2$. Therefore, $\pi(0.5s_1 + 0.5s_1', s_2) = -0.5s_2$. So $\pi(0.5s_1 + 0.5s_1', s_2) \ngeq \min\{\pi_1(s_1, s_2), \pi_1(s_1', s_2)\}$. So we do not have quasiconcavity in s_1.

Question 3

Suppose player 2 is using a uniform distribution $[0, 1]$.

$$\pi_1(s_1, s_2 \sim \text{uniform}[0,1]) = \int_0^{s_1} (1 - s_1)dx + \int_{s_1}^{1} -s_1 dx = 0$$

This is independent of s_1, so choosing from uniform distribution is indeed a NE.

Question 3

Now suppose we do not know the distribution.

$$\pi_i(s_i, f_j) = \int_0^{s_i} (1 - s_i) f_j(x) dx + \int_{s_i}^{1} -s_i f_j(x) dx$$

$$= \int_0^{s_i} f_j(x) dx - s_i \Rightarrow \text{ F.O.C} : f_j(s_i) = 1$$

So the distribution is indeed uniform. We cannot use Nash existence result since it was stated for finite games.

Question 4

$$\pi_i(q_i, q_{-i}) = (a - b\sum_{j=1}^{J} q_j)q_i - cq_i$$

Taking F.O.C and solving for the symmetric solution:

$$q_i^* = \frac{a-c}{b(J+1)}$$

$$Q^* = \frac{J}{J+1} \cdot \frac{a-c}{b}, \;\; \frac{\partial Q^*}{\partial J} > 0$$

$$P^* = a - \frac{J}{J+1}(a-c), \;\; \frac{\partial P^*}{\partial J} < 0$$

As $J \to \infty$, we get the competitive result.

Question 5

$$\max\ (p(q,t) - k_i)q_i^t$$

F.O.C gives us the results. From $\dfrac{q_1^0}{q_2^0} > 1$, we get $k_1 < k_2$.

From $\dfrac{q_1^1}{q_2^1} > \dfrac{q_1^0}{q_2^0}$, we get $p(q,1) < p(q,0)$.

Question 6

$\pi_i = (a - b(q_1 + q_2))q_i - c_i q_i$

It is easy to get the equilibrium of the game if both firms are producing a strictly positive amount. If $q_1^* < 0$, then $2c_1 - c_2 > a$ and only the firm with lower cost can be in the market and make positive profit.

$$NE = \begin{cases} (\frac{a + c_2 - 2c_1}{3b}, \frac{a + c_1 - 2c_2}{3b}) & a \geq 2c_1 - c_2 \\ (0, \frac{a - c_2}{2b}) & a < 2c_1 - c_2 \end{cases} \quad (1)$$

Question 6

$$\frac{\partial q_1^*}{\partial c_1} < 0, \;\; \frac{\partial q_2^*}{\partial c_1} > 0$$

$$\frac{\partial \pi_1^*}{\partial c_1} < 0, \;\; \frac{\partial \pi_2^*}{\partial c_1} > 0$$

Also, $P^* \uparrow$ leads to $Q^* \downarrow$.

Question 6

In Nash equilibrium, each firm maximizes its own profit:

$$\max_{q_j \geq 0} (p(q_j + Q^*_{-j}) - c_j)q_j$$

WLG, we can focus on the firms that produce a positive output.
From F.O.C we have $p'(Q^*)q_j^* + p(Q^*) - c_j \Rightarrow \pi_j^* = -p'(Q^*){q_j^*}^2$.
$\dfrac{\sum \pi_j^*}{Q^* p(Q^*)} = \dfrac{H}{\epsilon}$.