PS10-Solution

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Question 1-part a

Since we are looking at the pooling equilibrium, player U cannot figure out the type based on player I's signal. So he only looks at the expected value of the commodity which is 90. Player I will not bid more than 100 because for both types, same signal will be sent. If player I bids less than 90 and player U rejects, then player I can bid higher than 90 when the type is a and get higher payoff. So bidding less than 90 and player U rejecting is not an equilibrium. So player I bids $x \in \{90, 91, ..., 100\}$ and player U accepts. The beliefs of player U that supports this is equilibrium is when he sees a bid x, he assigns probability $1/2$ to state a and for any other bid, he assigns probability 1 to state a. If he assigns probability $1/2$ to state a regardless of the bid, then the only equilibrium would be offering 90 and accepting.

Question 1-part b

Since in this case player U learns the type, then if he rejects always in state a, player I can offer anything from the lowest to highest value to player U, so $x_a \in \{1, 2, .., 120\}$. For player U to accept in type b, player I will not offer anything less than 60 because player U will reject and player I could have offered 121, which player U accepts, and made a profit. If player I bids x_a in type b, he gets 0 profit while bidding x_b will get positive profit in state b. Also, he will not deviate in state a to bid x_b as long as the profit of non-deviation is higher: $100 - x_b \leq 0 \Rightarrow x_b \geq 100$. Note also that player U will accept anything with $x_b \geq 121$, and bidding more than 121 will decrease the payoff of player I. So the PBE will be $x_b \in \{100, 101, ..., 121\}$ and player U accepts. The belief is that if x_b is offered, then probability of state b is 1, for any other offer, this probability is zero.

Question 1-part c

It is weakly dominated for I to offer any $x_a > 100$. Given that, U "should" conclude upon seeing a defection $x > 100$, that b has occurred, and hence accept any $x \geq 101$. An offer of 101 is clearly better (for I) than any higher offer, so the "reasonable" equilibrium have $x_b \in \{100, 101\}$.

Question 2

If we have a SE, then it is sequentially rational and the beliefs are derived from Bayes rule when possible, so it is WPBE. For proving the other side, assume that we have only i types and construct the following perturbation of the WPBE:
$\alpha^n(x|t_i) = \hat{\alpha} - \dfrac{1}{n_i}$ and $\alpha^n(y|t_i) = \dfrac{1}{n_i}$ when $\hat{\alpha}(x|t_i) > 0$ and $\hat{\alpha}(y|t_i) = 0 \; \forall t_i \in T$. The the beliefs and the limits would be:

$$\beta^n(t_i|x) = \frac{p(t_i)(\hat{\alpha}(x|t_i) - \frac{1}{n_i})}{\sum_{t_j \in T} p(t_j)(\hat{\alpha}(x|t_i) - \frac{1}{n_j})} \Rightarrow \hat{\beta}(t_i|x) = \frac{p(t_i)\hat{\alpha}(x|t_i)}{\sum_{t_j \in T} p(t_j)\hat{\alpha}(x|t_i)}$$

Question 2

For y, we have:

$$\beta^n(t_i|y) = \frac{p(t_i) \cdot \frac{1}{n_i}}{\sum_{t_j \in T} p(t_j) \cdot \frac{1}{n_j}} \Rightarrow \hat{\beta}(t_i|y) = \frac{p(t_i)}{\sum_{t_j \in T} p(t_j)}$$

So we have consistency and from WPBE, sequential rationality.
So it is SE.

Question 3-part a

Firm B maximizes the expected profits:

$$\max_{\{q_{B\geq 0}\}} \ \pi(4 - q_A - q_B)q_B + (1-\pi)(3 - q_A - q_B)q_B$$

The first order condition implies:

$$q_B = \frac{3 - q_A + \pi}{2}$$

So q_B depends positively on π.

Question 3-part b

We have to check that incentives are compatible:

$$(4 - q_A(H) - q_B(H))q_A(H) \geq (4 - q_A(L) - q_B(L))q_A(L)$$

$$(3 - q_A(L) - q_B(L))q_A(L) \geq (3 - q_A(H) - q_B(H))q_A(H)$$

Adding the up, we have: $q_A(H) \geq q_A(L)$.

Question 3-part c

If they both know the demand, we have a Stackelberg game and the outputs would be $q_A(H) = 2$ and $q_A(L) = 1.5$. These come from optimization, so based on the strategy of player B, player A strictly better off with choosing $q_A(H) = 2$.

Question 3-part d

If we look at the separating equilibrium, then when player A sets $q_A(H)$, player B knows that the demand is high. Thus $\pi = 1$ and we have the same situation as part c, so $q_A(H) = 2$. For $q_A(L)$, if we look at the incentive, we want to discourage deviations. So if player A plays L when the state is H:

$$(4-2-1)2 = 2 \geq (4-q_A(L)-\frac{3-q_A(L)}{2})q_A(L) \Rightarrow (4-q_A(L))(1-q_A(L)) \geq 0$$

Here, $\pi = 0$ since the signal indicates the low type demand. From part b, we know that $q_A(L) \leq 2$. So we must have $q_A(L) \leq 1$. We cannot have $q_A(L) = 1.5$ since it is too high to trick firm B to cut output and derive up the price. Setting this output results in the profit of $\frac{21}{8} > 2$. So it cannot happen since it is not incentive compatible.

Question 3-part d

For the low demand, in order to prevent deviation we must have:

$$(3 - q_A(L) - \frac{3 - q_A(L)}{2})q_A(L) \geq \max_{\{q_A \geq 0\}} (3 - q_A - \frac{4 - q_A}{2})q_A$$

This gives us $q_A(L) \in [\frac{3-\sqrt{5}}{2}, \frac{3+\sqrt{5}}{2}]$. The previous condition together with this one implies $q_A(L) \in [\frac{3-\sqrt{5}}{2}, 1]$. The beliefs system that supports this as a PBE is:

$$\pi = 1 \text{ if } q_A = q_A(L),\ \pi = 0 \text{ otherwise}$$