PS1-Solution

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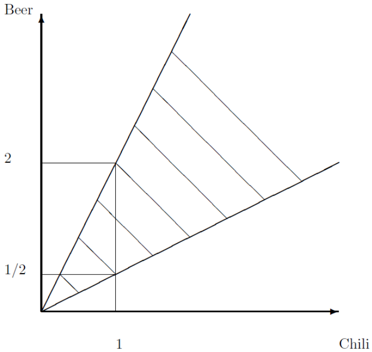
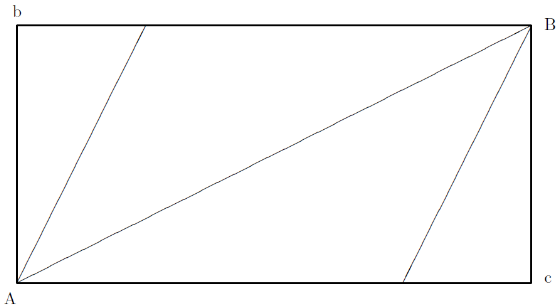
Question 1-Part a



$$C = \{(x, y) | x \geq 0,\ \frac{x}{2} \leq y \leq 2x\}$$

question 1-Part b



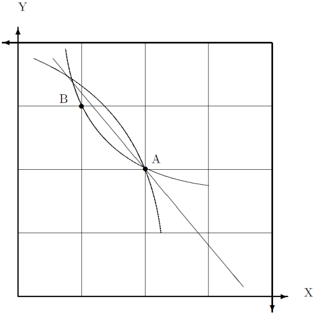
Question 1-Part c

Let $p_b > 0$ and $p_c = 0$. Suppose endowments are of te form $2w_b = w_c$. In this case, none can have more beer and they will not trade chili for beer.

Question 1-Part d

No!
With the consumption set being the positive orthant, both individuals will demand infinite amount (preferences are strictly increasing) for a good with zero price.

Question 2-Part a



In the figure above, given the price ratio, point A is a WE. Point B is Pareto imporovement for consumer B since it makes consumer 1 indifferent and consumer 2 strictly better off but it is not a WE.

Question 2-Part b

Yes!

Assume we have a WE $((p_{1^*}, p_2^*), (x_1^*, y_1^*), (x_2^*, y_2^*))$, which is not PO. Assume further that a new allocation Pareto dominates the WE: $((x_1, y_1), (x_2, y_2))$. Let good x be divisible. The WE implies:

$$p_1^* x_1^* + p_2^* y_1^* = p_1^* w_{1x} + p_2^* w_{1y}$$
$$p_1^* x_2^* + p_2^* y_2^* = p_1^* w_{2x} + p_2^* w_{2y}$$

For the PO allocation, suppose consumer 2 is strictly better off:

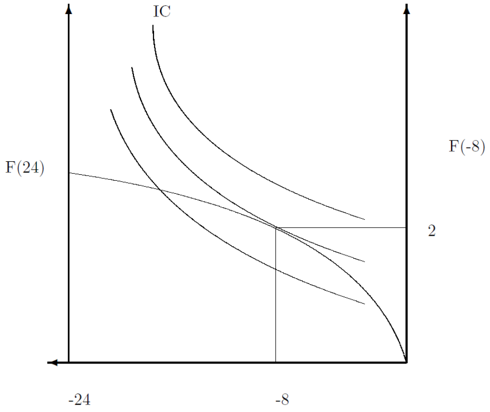
$$p_1^* x_2 + p_2^* y_2 > p_1^* x_2^* + p_2^* y_2^* = p_1^* w_{2x} + p_2^* w_{2y}$$

Summing up first and third equation:

$$p_1^*(x_1^* + x_2) + p_2^*(y_1^* + y_2) > p_1^*(w_{1x} + w_{2x}) + p_2^*(w_{1y} + w_{2y})$$

which violates feasibility!

Question 3-Part a(A)



$$U(F, L) = \log F + \log(24 + L), \;\; F = (-L)^{0.5}$$

Question 3-Part b(A)

PO allocations:

$$\max_{\{F,-L\}} \log F + \log(24-(-L)), \quad s.t.\ F = (-L)^{0.5}$$

This gives us $F = \sqrt{8},\ -L = 8$.

Question 3-Part c(A)

WE (setting price of F to one):

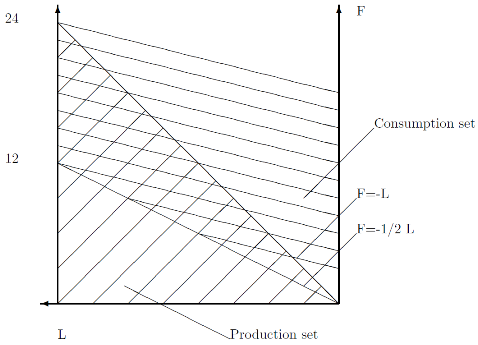
$$\max_{\{F,-L\}} \log F + \log(24-(-L)), \quad s.t.\ F = w(-L)+\pi$$

$$\max_{\{-L\}} (-L)^{0.5} - w(-L)$$

Second maximization gives us $w = \frac{0.5}{(-L)^{0.5}}$ and the resulting profit is: $\pi = 0.5(-L)^{0.5}$. The equilibrium wage and the profit will turn the constraint of the first problem into $F = (-L)^{0.5}$, which is turns this problem essentially into the PO allocation problem. Hence we have:

$$F_c = F_f = \sqrt{8}, L_c = L_f = -8, w = \frac{1}{\sqrt{32}}, p = 1$$

Question 3-Part a(B)



$$U(F, L) = \log F + L, \ \ F = CRS$$

Question 3-Part b(B)

PO allocations:

$$\max_{\{F,-L\}} \log F - (-L), \quad s.t.\ F = -L$$

This gives us: $F = 1,\ -L = 1.$

Question 3-Part c(B)

WE (setting price of F to one):

$$\max_{\{F,-L\}} \log F-(-L), \quad s.t.\ F=w(-L)+\pi$$

$$\max_{\{-L\}} (-L)-w(-L)$$

The WE is: $p=1, w=1, L_c=L_f=-1, F_c=F_f=1.$

Question 4

Proof by contradiction:
Suppose that B is not convex, then there exist $a, b \in B$, s.t. $x = \lambda a + (1-\lambda)b \notin B$, $\lambda \in [0,1]$. It means that we can find two points in B such that a convex combination of them is outside of the set B. Then there is a hyperplane separating B and $x : \exists p \in \mathbb{R}^N$ with $p \neq 0$ and $c \in \mathbb{R}$ s.t.
$p \cdot x > c,\ p \cdot y < c,\ \forall y \in B.$
Since $a, b \in B, p \cdot a < c, p \cdot b < c$, then
$\lambda p \cdot a + (1-\lambda)p \cdot b < c \Rightarrow p[\lambda a + (1-\lambda)b] < c$, which is $p \cdot x < c$.
So, B is convex.