Lindahl Equilibrium

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February 28, 2017

Definition of an Externality

An externality is present whenever the well-being of a consumer or the production possibilities of a firm are directly affected by the actions of another agent in the economy.

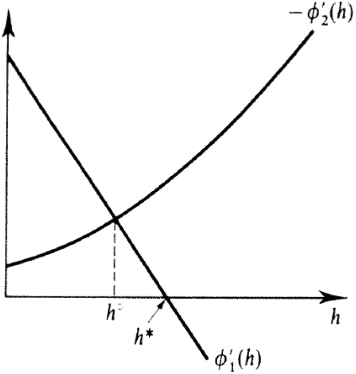
Directly means to exclude any effects that are mediated by prices.

Modeling Externality

- two consumers indexed by $i = 1, 2$, price takers with wealth w_i
- preferences are over consumptions of L goods and the actions of consumer 1: $u_i(x_{1i}, ...x_{Li}, h)$
- $$v_i(p, w_i, h) = \max_{x_i \geq 0} \; u_i(x_i, h), \qquad \text{subject to } p \cdot x_i \leq w_i$$
- assume quasilinear preferences with respect to a numeraire commodity: $v_i(p, w_i, h) = \phi_i(p, h) + w_i$
- $\phi_i(\cdot)$ is twice differentiable and $\phi_i(\cdot) > 0, \;\; \phi_i''(\cdot) < 0$

Nonoptimality of the Competitive Outcome

- assuming interior solutions, consumer 1 chooses the optimal level of h: $\phi_1'(h^*) = 0$.
- the pareto optimal allocation must maximize the joint surplus of the two consumers: $\phi_1'(\mathring{h}) = -\phi_2'(\mathring{h})$



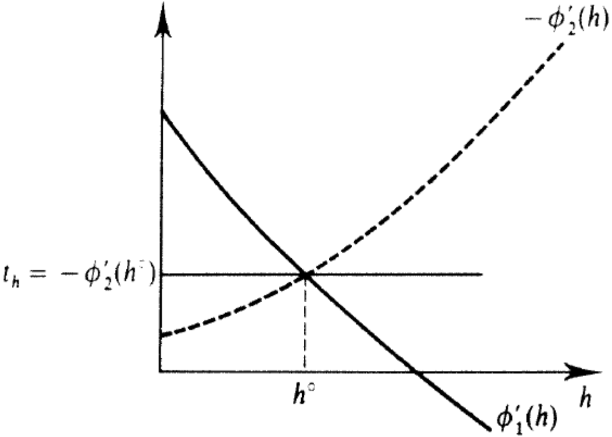
Solutions- Quotas and Taxes

- Government can make consumer 1 to choose $\mathring{h}$.
- *Pigouvian taxation*: suppose government imposes a tax of $t_h = -\phi_2'(\mathring{h})$ per unit of h.

$$\max_{h \geq 0} \ \phi_1(h) - t_h h \Rightarrow \phi_1'(h^*) = t_h = -\phi_2'(\mathring{h}) \Rightarrow h^* = \mathring{h}$$

- The optimality-restoring tax is exactly equal to the *marginal externality* at the optimal solution.
- Consumer 1 *internalizes* the externality that she imposes on consumer 2.

Solutions- Quotas and Taxes



Solutions- Quotas and Taxes

- Optimality can be achieved by either taxing the externality or subsidizing its reduction.
- It is essential to tax the externality-producing activity directly.
- The tax/subsidy or quota approaches are equally effective, but government must know a great deal of information.

Solutions- Bargaining

- We have enforceable property rights regarding the production of externality.
- Suppose we assign the right to an "externality-free" environment to consumer 2
- Consumer 2 make a take-it-or-leave-it offer to consumer 1, demanding payment T and level h

$$\max_{\{T,h\geq 0\}} \phi_2(h) + T, \quad \text{subject to } \phi_1(h) - T \geq \phi_1(0)$$

The constraint is always binging: $T = \phi_1(h) - \phi_1(0)$

$$\max_{h\geq 0} \phi_2(h) + \phi_1(h) - \phi_1(0) \Rightarrow \phi_1'(\mathring{h}) = -\phi_2'(\mathring{h})$$

Solutions- Bargaining

- The assignment of property right is irrelevant to the optimal allocation, just the distribution of income changes.
- Exercise: assign property right of producing the externality to consumer 1 and find the optimal level of h.

Coase (1960)

If trade of the externality can occur, then bargaining will lead to an efficient outcome no matter how the property rights are allocated.

- The existence of both well-defined and enforceable property rights is essential for this type of bargaining to occur.
- The government needs to know very little information.

Missing Markets

- Assume the externality-free environment right is with consumer 2. Let p_h denotes the price of the right to engage in one unit of the activity.
- $$\max_{h_1 \geq 0} \ \phi_1(h_1) - p_h h_1 \Rightarrow \phi_1'(h_1) = p_h$$
  $$\max_{h_2 \geq 0} \ \phi_2(h_2) + p_h h_2 \Rightarrow -\phi_2'(h_2) = p_h$$
  $$\Rightarrow \phi_1'(h^{**}) = -\phi_2'(h^{**}) = p_h$$
- Externalities can be seen as being inherently tied to the absence of certain competitive markets.
- Price taking assumption in the above example is questionable.

Public Goods

Table: 1. Types of Goods

	Excludable	Non-Excludable
Rivalrous	Private Goods	Common-pool Resources
Non-Rivalrous	Club Goods	Public Goods

Private Goods: food, clothing, cars, parking spaces
Common-pool Resources: fish stocks, timber, coal
Club Goods: cinemas, private parks, satellite television
Public goods: free-to-air television, air, national defense

Conditions for Pareto Optimality

- I consumers, L traded goods and one public good
- quantity of public good has no effect on the price of traded goods, preferences are quasilinear with respect to the same traded numeraire good
- derived utility of consumer i for public good: $\phi_i(x), \; \phi_i'(\cdot) > 0 \; \phi_i''(\cdot) < 0.$
- provision of public good is costly: $c(q), \; c'(\cdot) > 0, \; c''(\cdot) > 0$
- Pareto optimal allocations:

$$\max_{q \geq 0} \sum_{i=1}^{I} \phi_i(q) - c(q) \Rightarrow \sum_{i=1}^{I} \phi_i'(\mathring{q}) = c'(\mathring{q})$$

- *the sum of consumers' marginal benefits is set to the marginal cost*

Inefficiency of Private Provision of Public Goods

- a market for public good exists and consumers buy x_i amount at price p.
- a single firm produces public good with cost function $c(q)$, and let $x = \sum_{i=1}^{I} x_i$
- consumer i

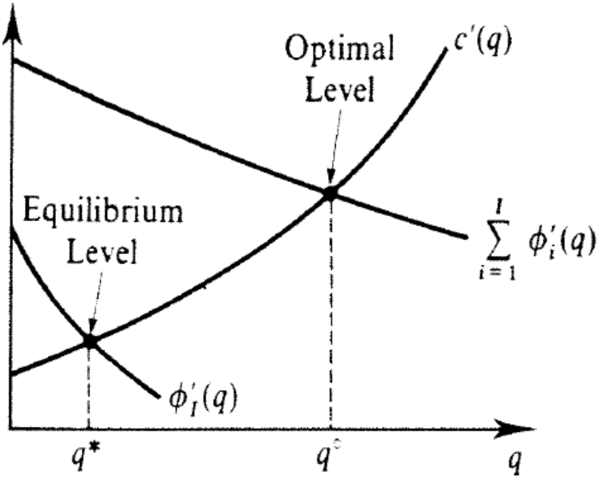
$$\max_{x_i \geq 0} \ \phi_i(x_i + \sum_{k \neq i} x_k^*) - px_i \Rightarrow \phi_i'(x_i^* + \sum_{k \neq i} x_k^*) = p = \phi_i'(x^*)$$

- Firm

$$\max_{q \geq 0} \ pq - c(q) \Rightarrow c'(q^*) = p$$

- in equilibrium:
  $q^* = x^* \Rightarrow$ whenever $I > 1$ $\sum_{i=1}^{I} \phi_i'(q^*) > c'(q^*)$

Inefficiency of Private Provision of Public Goods



Inefficiency of Private Provision of Public Goods

Free Rider Problem:

- Consumers can take advantage of public goods without contributing sufficiently to their creation.
- If private organizations do not reap all the benefits of a public good which they have produced, their incentives to produce it voluntarily might be insufficient.

Proposed Solution: Lindahl Equilibrium (1919)

Lindahl Equilibrium

- Suppose there exists a market for each consumer's consumption of the public good.
- Each consumer's consumption is viewed as a distinct commodity, so for consumer i, the price is p_i.
- consumer i

$$\max_{x_i \geq 0} \; \phi_i(x_i + \sum_{k \neq i} x_k^*) - p_i x_i \Rightarrow \phi_i'(x^*) = p_i$$

- firm

$$\max_{q \geq 0} \; \left(\sum_{i=1}^{I} p_i q \right) - c(q) \Rightarrow \sum_{i=1}^{I} p_i = c'(q^*)$$

- in equilibrium: $x^* = q^* \Rightarrow \sum_{i=1}^{I} \phi_i'(q^*) = c'(q^*)$

Environment

A *pure public good economy* (with one public good and one private good) is a list $E = \langle N; (w_i)_{i\in N}; (u_i)_{i\in N}; c\rangle$ consisting of the following elements:

- N is a non-empty finite set of consumers.
- Each consumer $i \in N$ has an initial endowment w_i of the private good, and a utility function $u_i(x, y_i)$ for consumption of amounts x of the public good and y_i of the private good.
- There is a single producer of the public good and c(x) is the cost in terms of private good for producing an amount x of the public good.

Environment

- each consumer and producer takes prices of all goods as given, maximizes utility and profits respectively
- each consumer is assumed to face a personalized price for units of the public good and this price can be different for each consumer
- exogenously given distribution rule determines the distribution of possible positive profits among consumers
  - distribution rule: $d = (d_i)_{i \in N}$, $\sum_{i \in N} d_i = 1$
- all consumers demand the same level of public good

Equilibrium

A d-Lindahl equilibrium consists of a vector of personalized prices $p^* = (p_i^*)_{i \in N}$[1], an amount of public good x^*, and amounts of private good $(y_i^*)_{i \in N}$ such that:

- x^* is a solution to

$$\max_{x \geq 0} \ \Big(\sum_{j \in N} p_j^*\Big)x - c(x)$$

- For each $i \in N$, (x^*, y_i^*) is a solution to

$$\max_{(x,y_i) \geq 0} \ u_i(x, y_i)$$

$$\text{subject to } p_i^* x + y_i \leq w_i + d_i\Big(\Big(\sum_{j \in N} p_j^*\Big)x^* - c(x^*)\Big)$$

[1] Price of the private good is normalized to one.

Equilibrium

If we impose restrictions of preferences, i.e. continuity, convexity, local nonsatiation, and production, i.e. monotonicity and concavity, then we can use theorems in general equilibrium so far and conclude that the first and second welfare theorems work. Thus, we have a market based solution for the problem of free riding by charging consumers for use of the public good.

Example 1

- $N = \{1, 2\}$ $w_1 = w_2 = 8$.
- $u_1(x, y_1) = x^{1/4} y_1^{3/4}$
- $u_2(x, y_2) = x^{3/4} y_2^{1/4}$
- $c(x) = 2x$

Problem of the producer:

$$\max_{x \geq 0} \ (p_1^* + p_2^*)x - 2x$$

This gives us $p_1^* + p_2^* = 2$. Profits are zero, so no need to worry about distribution. Utility functions are strictly increasing, so the budget constraints bind: $p_1^* x + y_1 = 8 \Rightarrow y_1 = 8 - p_1^* x$.

Example 1

Substituting into the utility function for consumer 1, we get:

$$x = \frac{2}{p_1^*}$$

Similarly for consumer 2:

$$x = \frac{6}{p_2^*}$$

These demands are equal in equilibrium, which together with $p_1^* + p_2^* = 2$ gives us the solution:

$$p_1^* = 0.5,\ p_2^* = 1.5,\ x^* = 4,\ y_1^* = 6,\ y_2^* = 2$$

Exercise

In general, if the public good production exhibits constant returns to scale, profit maximization implies that, in equilibrium, the public good is only produced if the sum of the personalized prices is equal to the constant marginal cost.
Try a formal proof.

Example 2

- $N = \{1, 2\}$ $w_1 = 4$, $w_2 = 6$.
- $u_1(x, y_1) = x + y_1$
- $u_2(x, y_2) = 3x + y_2$
- $c(x) = x^2$

Problem of the producer:

$$\max_{x \geq 0} (p_1^* + p_2^*)x - x^2 \Rightarrow x^* = \frac{p_1^* + p_2^*}{2}$$

$$\Rightarrow \pi = \left(\frac{p_1^* + p_2^*}{2}\right)^2$$

Here, we need a distribution rule: $d = (d_1, d_2)$, $d_1 + d_2 = 1$.

Example 2

- For consumer 1's utility-maximization problem to have an interior solution, the consumer's budget line needs to have the same slope as any of her indifference curves: $p_1^* = 1$.
- $p_2^* = 3$.
- This leads to $x^* = 2$, profit of the firm $\pi = 4$.
- $y_1^* = 4 - 2 + 4d_1 = 2 + 4d_1$.
- $y_1^* = 4d_2$

In general, we do not have uniqueness, the distribution rule changes the equilibrium.

Caveats

- Price taking assumption is not satisfied.
- Inherent problem of public goods is unsolved: private information
- Need of exclusion for those who do not pay their prices.

References

- MWG: Chapter 11 and pages 568-70.
- Kreps: Pages 381-2
- https://papers.ssrn.com/sol3/papers2.cfm?abstract_id=2266604